# Topologically Stable Z - Strings in the Supersymmetric Standard Model

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**Abstract**

We show that the minimal supergravity extension of the standard model automatically contains topologically stable electroweak strings if the hidden sector is invariant under the exact R-symmetry. These defects appear in the form of the semiglobal R-strings, which necessarily carry $Z$-flux inside their core. This result is independent from the particular structure of the hidden sector.

Discussed strings differ fundamentally from the embedded $Z$-strings. If R-symmetry is explicitly broken (e.g. due to an anomaly), the decay of the semiglobal strings may have important implications for the baryogenesis.

## 1. Introduction

According to the present understanding of cosmology, spontaneous breaking of some gauge symmetry $G_l$ (down to its subgroup $H_l$) may lead to the formation of the topologically stable vacuum defects. As is well known, if the first homotopy group of the vacuum manifold is nontrivial $\pi_1(G_l/H_l) \neq 1$, these stable defects are strings. Strings carry a topologically conserved gauge flux in their core. Thus the above condition is usually considered as a necessary condition for the existence of the topologically stable flux inside the defect.

At present the only known example of the spontaneously broken gauge symmetry in nature is $SU(2) \otimes U(1)$ electroweak group. Therefore, it is very important to know whether there might exist topologically stable defects that carry electroweak flux. Since $\pi_1[SU(2) \otimes U(1)/U(1)_{EM}]$ is trivial, one would normally expect that there can not be any topologically conserved electroweak flux unless $SU(2) \otimes U(1)$ is embedded in some larger local symmetry group (e.g. in grand unification symmetry). However, we have shown recently that this assumption is false [1]; topologically stable electroweak flux can easily exist in the theory provided the global symmetry group $G_{gl}$ is larger by at least the spontaneously broken $U(1)_{gl}$ factor under which electroweak Higgs doublet transforms nontrivialy. Thus, it turns out that the condition $\pi_1(G_l/H_l) = 1$ does not contradict necessarily the topological stability of the flux provided the actual exact global symmetry $G_{gl}$ (which covers $G_l$) broken down to $H_{gl}$ (covering $H_l$) satisfies $\pi_1(G_{gl}/H_{gl}) = Z$.

As was shown in [1], the minimal extension of the standard model in which

topologically stable electroweak strings are presented is an $SU(2) \otimes U(1)$ gauge theory with two Higgs doublets and an $U(1)_{gl}$ global symmetry under which doublets carry different charges. These strings should not be confused with the embedded $Z$-strings [2] or/and the semilocal ones [3]. Embedded string solutions are always topologically unstable (but can be stable under small perturbations in some range of parameters). It is impossible to obtain our solution from the embedded (or semilocal) string by continuous change of the parameters (preserving $SU(2) \otimes U(1)$-symmetry). The reason is simple, by definition embedded (or semilocal) strings are just usual Nielsen-Olesen [4] type vorteces, but embedded into a larger local (or global) $SU(2) \otimes U(1)$ symmetry. Therefore in the case of embedded $Z$-strings all vacuum expectation values (VEVs), independently of the number of Higgs doublets, wind by *one and the same* $U(1)_Z$-gauge transformation. In contrast, the crucial point for the existence of the topologically stable Z-strings is that the phases of the two doublets should wind $differently$ (since the transformation around the string is never a pure gauge) and this guarantees the topological stability of the $Z$-flux. To avoid possible confusion we will call our strings "semiglobal" in order to indicate the role played by the global symmetry.

Of course, the $U(1)_{gl}$ symmetry, if it is exact, has to be broken at some very high scale (exceeding $10^9 GeV$ or so [5]) by some $SU(2) \otimes U(1)$-singlet VEV, otherwise the resulting Goldstone bosons (which inevitably couple to matter fermions) can not obey astrophysical constraints. Thus, the minimal realistic Higgs sector allowing topologically stable electroweak strings includes two doublets and at least one gauge singlet with a large VEV. Naturally, we would like to know which are the physically important extensions

of the minimal electroweak model that allow stable strings.

We find it rather interesting that conventional $N = 1$ supergravity extensions of the standard model [6] automatically fit in this class of theories. Due to supersymmetry (SUSY), they contain at least one pair of Higgs doublets and a number (at least one) of $SU(2) \otimes U(1)$-singlets with a large VEVs in the "hidden" sector (which is responsible for SUSY breaking). By definition, hidden sector Higgs fields are allowed to have only gravitational strength interactions (suppressed by powers of Planck mass $M_{pl}$) with an "observable" sector and thus should be singlets under "observable" gauge symmetries.

In the present paper we study the existence of the semiglobal electroweak strings in the locally supersymmetric standard model and point out some of their possible cosmological consequences. It turns out that in the minimal case (which does not assume any extra gauge singlets in the observable sector) the sufficient condition for the existence of these defects is an exact (or approximate) $R$-symmetry in the hidden sector.

# 2. Topologically stable flux

In this section we will briefly recall the mechanism of Ref. [1] leading to the topological stability of the Z-flux and indicate more explicitly the difference of this solution with respect to embedded or semilocal strings. Consider an $SU(2) \otimes U(1)$-theory with two Higgs doublets $H$ and $\bar{H}$ with the opposite hypercharges and an extra global symmetry $U(1)_{gl}$ under which their charges are *not* opposite (we use the usual supersymmetric convention of a doublet and an antidoublet). As we said before, $U(1)_{gl}$ has to be broken at some

high scale $M_{gl}$ with a VEV of the gauge singlet scalar $S$. The relative $U(1)_{gl}$-charges of the fields $H$, $\bar{H}$ and $S$ are fixed from the explicitly phase dependent couplings in the scalar potential. In the renormalizable theory there are two alternative terms $SH\bar{H}$ or $S^2H\bar{H}$ (+ h.c.). Let us for definiteness choose the trilinear one, which in terms of the $U(1)_{EM}$-invariant VEVs can be written in the form

$$V_{phase} = \mu v \bar{v} s \cos\left(\theta_S - \theta\right) \tag{1}$$

where $\langle H_0 \rangle = v e^{i\theta_H}$, $\langle \bar{H}_0 \rangle = \bar{v} e^{-i\theta_{\bar{H}}}$ and $\langle S \rangle = s e^{i\theta_S}$ are the VEVs of the electrically neutral fields and $\theta = \theta_{\bar{H}} - \theta_H$.Furthermore, $\mu$ is a mass parameter which we will take to be real and negative. Note, that in order to have a correct value for the weak scale ($m_W$), $\mu$ has to be of order $m_W^2/M_{gl}$. This means that for $M_{gl}$ large, $\mu$ should be very small. However, this choice is "technically" natural, since the limit $\mu = 0$ enlarges the symmetry by an extra global $U(1)$.

The VEV of the $S$ field breaks $U(1)_{gl}$ and forms the string. The phase $\theta_S$ winds by $2\pi n$ (n is an integer) around the string and so does $\theta$, since $\theta_S$ and $\theta$ are locally correlated through the term (1), which requires $\theta_S = \theta$. Therefore we have a topological constraint

$$\oint \partial_\mu \theta dx^\mu = 2\pi n \tag{2}$$

where the integral is taken along the path that encloses the string at infinity. The key point is that for the VEVs to be single valued, each of the phases $\theta_H$ and $\theta_{\bar{H}}$ has to wind by $2\pi$-integer. So we have

$$\oint \partial_\mu \theta_i dx^\mu = 2\pi n_i \tag{3}$$

with $n_i$ ($i = H, \bar{H}$) integers that must satisfy a topologically invariant condition

$$n_{\bar{H}} - n_H = n \tag{4}$$

Let us take n=1 (corresponding to the minimal global string). The necessary existence of the flux can be easily seen from the equation of motion for the $Z$-boson which at the spatial infinity (where all field strengths vanish) has the form:

$$Z_\mu = \frac{\cos\theta_W}{g} \frac{v^2 \partial_\mu \theta_H + \bar{v}^2 \partial_\mu \theta_{\bar{H}}}{v^2 + \bar{v}^2} \tag{5}$$

Now taking the integral around the same path and using the condition (3) and (4) we immediately obtain:

$$Z - flux = \frac{\cos\theta_W}{g} \frac{(v^2 + \bar{v}^2) n_H + \bar{v}^2}{v^2 + \bar{v}^2} \tag{6}$$

Obviously, for the minimal string characterized by n=1 the right hand side of this equation can never vanish, provided $v, \bar{v}$ are nonzero. Thus there is always the $Z$-flux in the core of the string. This flux is rather unusual. In contrast with an ordinary gauge string flux it is not a integer so that it can never compensate completely a logarithmic divergence of the gradient energy at the infinity. These strings exhibit properties of both global and local strings; they are semiglobal. Furthermore, although the flux by itself has no

topological origin, it is topologically stable since the strings are topologically stable and there can be no string without the flux.

It is important to understand the fundamental difference between our semiglobal $Z$-strings and the embedded or/and semilocal ones. As we have pointed out earlier the key difference comes from the fact that for the embedded (or semilocal) defect the VEVs wind by usual local $U(1)_Z$-transformation. This automatically implies that the phases of all the Higgs doublets (that carry same $SU(2) \otimes U(1)$- quantum numbers) should wind in the same way. In the sharp contrast as we have shown, the very existence of the topologically stable semiglobal strings is just based on the topological constraint (2) which implies that phases of doublets must wind differently ($n_H - n_{\bar{H}} = integer$). In particular, in the n=1 case one expects that one of the doublets does not wind at all, so that $U(1)_Z$ gauge symmetry is not necessarily restored in the core.

In the case of the semilocal strings $\pi_1(G_l/H_l)$ is nontrivial and formally there is a topological flux in the gauge sector. This flux however is never topologically stable since $\pi_1(G_{gl}/H_{gl})$ is trivial and it costs a finite energy for the flux to be spread out. In the case of the semiglobal strings situation is just opposite: there is no topological flux in the gauge sector, but nevertheless the flux is topologically stable due to the stability of the Higgs configuration.

# 3. Supergravity extension

In the minimal N=1 supergravity extensions of the standard model one assumes the superpotential of the form

$$f = h(S_\alpha) + w(Y_i) \tag{7}$$

where $h$ and $w$ are polynomials in the superfields $S_\alpha$ and $Y_i$ belonging to a hidden and observable (quark, lepton, Higgs) sectors respectively. It is assumed that there is no coupling among $S$ and $Y$ fields in the superpotential. Therefore, this two sectors of the theory communicate only through the gravitational strength interactions in the potential. The most general renormalizable form of $w$ compatible with $SU(2) \otimes U(1)$ is

$$w = \mu H\bar{H} + g_u HQu^c + g_d \bar{H}Qd^c + g_l \bar{H}Le^c + (HL + QLd^c + LLe^c + u^c d^c d^c) \tag{8}$$

where $H$,$\bar{H}$ are Higgs doublets and $Q$,$u^c$,$d^c$,$L$,$e^c$ are quark and lepton superfields respectively; $g_{u,d,l}$ are "Yukawa" coupling constants (family indices are suppressed) and $\mu$ is a mass parameter which has to be of order $m_W$ for the correct electroweak symmetry breaking. The terms in the bracket are usually forbidden by the matter parity symmetry, since if all present they lead to too rapid proton decay. We keep them here only to emphasize the generality of our argument, the reader can choose the desired combination consistent with phenomenological constraints.

The important point is that the form of the observable superpotential automatically respects continuous global R-symmetry $U(1)_R$ under which for example the R-charges are $R_H = R_{\bar{H}} = R_L = 1$, $R_Q = 1/3$, $R_{u^c} = R_{d^c} = 2/3$ nad $R_{e^c} = 0$ (R-charge of the superpotential is normalized as usual to be 2). Note that under this R-charge assignment the form of $w$ given by (7)

becomes most general to all orders (including all possible nonrenormalizable operators).

We do not want to debate here a currently popular dilemma whether all global symmetries are necessarily broken by Planck scale induced operators. We simply consider both possibilities. So let us assume for a moment that exact R-symmetry is respected to all orders in $M_{pl}$. Therefore, for the superpotential (6), the R-transformation will be a valid symmetry if it is respected by the hidden sector. If this is the case, R-symmetry will be inevitably broken at some high scale $M_R$ (not below the scale $M_S$ at which SUSY gets broken). Breaking occurs due to nonzero VEV of the superpotential $\langle h \rangle$ (which can never vanish in the nonsupersymmetric minimum with zero cosmological constant) and due to the VEVs of scalars carrying nonzero R-charges. Spontaneous breaking of the R-symmetry forms global R-strings. Effects of the above breaking on the observable sector can be viewed from the effective low energy potential of the observable fields which has the following well known form [7]

$$V = \sum_i |\frac{\partial W}{\partial Y_i}|^2 + m_g^* A W^{(3)} + m_g^* B W^{(2)} + h.c. + \sum_i |m_g|^2 |Y_i|^2 + (D-terms) \quad (9)$$

Here $W(Y) = w(Y)\exp(2^{1/2}|S_\alpha|^2/M_{Pl})$ is a redefined low energy superpotential and $W^{(2)}$ and $W^{(3)}$ are its bilinear and trilinear (in $Y_i$) pieces respectively. $A$ and $B$ are numbers related to the details of the hidden sector.

The message about the R-symmetry breaking from the hidden sector is carried by the complex parameter $m_g$ whose absolute value is the gravitino mass. It so happens that $m_g$ sets the SUSY breaking scale in the visible

world. Explicit form of $m_g$ is

$$m_g = 8\pi\langle h\rangle M_{Pl}^{-2} \exp\left(2^{1/2}|S_\alpha|^2/M_{Pl}^2\right) \tag{10}$$

For us the important thing about $m_g$ (and $\langle h\rangle$) is that its phase $\theta_h$ winds by $2\pi n$ (with $n = integer$) around the R-string. This is clear, since $\langle h\rangle$ is polynomial in condensates $\langle S_\alpha\rangle$ which must be single valued. So the phase dependent coupling in (9) has the following form (for definiteness $B$ is taken negative):

$$V_{phase} = -|Bm_g\mu H_o\bar{H}_o| \cos\left(\theta_h - \theta\right) \tag{11}$$

As before $\theta = \theta_H - \theta_{\bar{H}}$,and since $\Delta\theta_h = 2\pi n$ around the string, arguments of Sec.2 are automatically valid. Thus, we conclude that topologically stable $Z$-flux gets trapped inside the R-string as soon as doublets $H$ and $\bar{H}$ pick up nonzero VEVs. This result is independent from the detailed structure of the hidden sector and the particular mechanism of the R-symmetry breaking.

# 4. Discussion on R-symmetry breaking

In general, the models with spontaneously broken exact R-symmetry in the hidden sector may face following difficulties: (1) Cancellation of the cosmological constant and/or (2) high scale of R- symmetry breaking which may be cosmologically problematic if $U(1)_R$ is anomalous.

(1) The first problem is related to the absence of the additive constant in the superpotential (unless it is dynamically generated) which makes it difficult to adjust the cosmological term to zero. However, the cancellation

of this term can in principle be achieved by means of adjusting the Kahler potential. An alternative possibility is to include some strongly coupled gauge interaction that breaks R-symmetry dynamically (as it is a case in the models with gaugino condensation [8]).

(2) The second difficulty may result from the high scale of R-symmetry breaking. Typically the scalar VEVs in the hidden sector are of order $M_{Pl}$ and can induce $U(1)_R$-breaking at a very high scale. Large $M_R$ can be cosmologically problematic if R-symmetry is anomalous. This is the case for instance in our example of the minimal SUSY standard model in which R-transformation acts on the quarks as an anomalous Peccei-Quinn symmetry $U(1)_{PQ}$ [9] and therefore should be broken at a scale $10^{10} - 10^{12} GeV$ [5]. As was stressed in the literature (e.g. see [5]), this value fits precisely the SUSY breaking scale. Unfortunately, to use R-symmetry for the solution of the strong CP-problem (without imposing additional $U(1)_{PQ}$-symmetries) seems to be problematic, since $M_R \sim M_S$ leads to the problem of nonzero cosmological constant. This difficulty comes from the general constraint [10] which says that the survival of any exact continuous R-symmetry to scales below $M_R \sim M_S^{2/3} M_{Pl}^{1/3}$ is incompatible with a zero cosmological constant. Namely, if a continuous R-symmetry is broken by the nonzero VEV of the superpotential $h$, then the the cancellation of the cosmological term implies that at the minimum

$$\langle h \rangle \sim M_{Pl} \langle F_S \rangle = M_{Pl} M_S^2 \tag{12}$$

where $\langle F_S \rangle$ is a VEV of the F-term that breaks supersymmetry. Thus it

looks likely that in theories with a spontaneously broken R-symmetry in the hidden sector either R-symmetry should be exact (nonanomalous) or if it is approximate, should be explicitly broken by a sufficiently large amount.

If R-symmetry is anomalous (acts as PQ-symmetry), it should be explicitly broken also by some other interaction which gives large enough mass to the would be R-axion and makes it compatible with cosmology. This explicit breaking can be induced for example by higher dimensional Planck scale operators (if we assume that supergravity does not respects R-symmetry) or/and by some additional metacolor sector which breaks $U(1)_R$ through the anomaly. Resulting semiglobal R-strings are no more topologically stable below the scale of the metacolor phase transition $M_{mc}$ and form boundaries of domain walls. The isolated string bounding infinite planar wall is stable for all practical purposes, since the probability of the hole formation is exponentially suppressed by the ratio $M_R/m_a$ where $m_a$ is a mass of would be R-axion. These structures will then, in the ususal manner, decay through the collapse [11].

# 5. Implications for Baryogenesis

Recently, there were some speculations about the possible role of the unstable vacuum defects in the baryogenesis. In particular in Ref. [12] it was argued that such a role can be played by the collapsing loops of the embedded $Z$-strings. The crucial point however is that to be relevant for the baryogenesis, embedded strings should be at least quasistable in order to survive to scales below the electroweak phase transition. Unfortunately, this is not the case in

the minimal standard model with a single Higgs doublet [13]. The authors of Ref.[12] assumed that embedded $Z$-strings might be stable in some realistic extensions of the minimal scheme, in particular in the two Higgs doublet model, which in the light of the very recent analysis [14] is doubtful. On the other hand, as we have shown, R-strings in the SUSY standard model are topologically stable electroweak $Z$-strings. But, as we said, if R-symmetry is anomalous it should be explicitly broken by sufficiently large amount either by gravity or by some strongly coupled metacolor sector. In such a case resulting $Z$-strings are no more topologically stable below the scale $M_{mc}$ where metainstanton (gravitational or some other) effects become important. But instability is just what one needs for the baryogenesis if the scale $M_{mc}$ is sufficiently low so that strings can survive below $m_W$. Below this scale (while still stable under tunneling) the string network will tend to decay through the collapse and may produce a certain baryon to entropy ratio. The quantitative analysis of this process depends on the detailed mechanism of R-symmetry breaking and is beyond the scope of the present paper.


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